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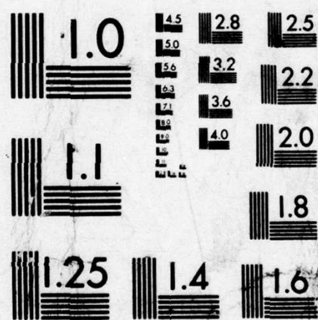
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METHODS FOR COMPUTING MAGNUS EFFECTS ON ARTILLERY PROJECTILES

Prepared by

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Davis, California 95616

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USA BALLISTIC RESEARCH LABORATORIES
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This report describes the research effort which was undertaken to extend the capability of numerical techniques developed for computing Magnus effects over right circular cones to artillery projectile shapes of interest to the Army. The changes involved: (1) introduction of longitudinal pressure gradient effects in the boundary layer equations; (2) introduction of turbulent transport properties into the boundary layer equations; (3) modification of the inviscid flow computation for a more general body configuration; and (4) modification of the technique for computing the three dimensional displacement surface.

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I. INTRODUCTION

This report describes research work whose aim was to extend a Magnus force calculation method over right circular cones to artillery projectiles. Previously, a method of calculating the Magnus force over a spinning right circular cone had been developed in references 1 and 2, and this method was restricted to laminar flow. In order for the methods developed to be extended to artillery projectiles, some significant changes had to be made. A listing of these changes will now be given.

(1) Axial pressure gradients were introduced into the boundary layer equations and calculation methods developed to take account of the curvature of the surface of an artillery type projectile.

(2) Turbulent transport properties were introduced into the boundary layer equations, since most realistic applications involve turbulent boundary layer flows. With the use of turbulent transport, a turbulence model had to be chosen, and also new methods had to be developed to calculate turbulent flow. The major new method involved the development of a coordinate transformation to resolve the peculiar structure of turbulent boundary layers with an efficient number of numerical node points.

(3) The calculation of the Magnus force had to be modified to reflect the change in geometry from a cone to an artillery projectile. This involved changing the method by which the boundary layer forces were integrated and changes in the calculation procedure for the three-dimensional boundary layer thickness.

(4) An additional study was carried out to assess the feasibility of using boundary region methods to calculate the viscous flow near the body. This study did not go past the stage of a derivation of the boundary region equations for the artillery projectile geometry. A detailed description of each of these modifications will be given.

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1. *H. A. Dwyer, "Three Dimensional Flow Studies Over a Spinning Cone at Angle of Attack," BRL Contract Report No. 137, U.S. Army Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland, February 1974. AD 774795.*

2. *H. A. Dwyer and B. R. Sanders, "Magnus Forces on Spinning Supersonic Cones, Part I: The Boundary Layer," AIAA Journal, Vol. 14, No. 4, April 1976, pp. 498-504.*

II. AXIAL PRESSURE GRADIENTS

For compressible boundary layer flow over an axisymmetric body, such as an artillery shell, defined by a radius of curvature $r(x)$, the three-dimensional boundary layer equations can be written as

$$\rho\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{w}{r}\frac{\partial u}{\partial \phi}-\frac{r'}{r}w^2\right]=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)\quad\text{(x--Momentum)}$$

$$\rho\left[u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+\frac{w}{r}\frac{\partial w}{\partial \phi}+\frac{r'}{r}uw\right]=-\frac{1}{r}\frac{\partial p}{\partial \phi}+\frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right)\quad(\phi\text{-- Momentum})$$

$$\rho c_p\left[u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\frac{w}{r}\frac{\partial T}{\partial \phi}\right]=u\frac{\partial p}{\partial x}+\frac{w}{r}\frac{\partial p}{\partial \phi}+\mu\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)\quad\text{(Energy)}$$

$$\frac{\partial}{\partial x}(\rho r u)+\frac{\partial}{\partial y}(\rho r v)+\frac{1}{r}\frac{\partial}{\partial \phi}(\rho r w)=0\ .\quad\text{(Continuity)}$$

The numerical solution of these equations can be considerably improved in accuracy and speed by transforming coordinates. The coordinate transformations are

$$\xi=\int_0^x r^2\,dx\qquad \overline{\phi}=\phi\qquad n=\left(\frac{p_\infty}{p_1}\right)^{\frac{1}{2}}\left(\frac{\rho_\infty u_\infty}{2u_\infty\xi}\right)^{\frac{1}{2}}\int_0^y\frac{\rho}{\rho_\infty}\,r\,dy\ .$$

In terms of these new independent variables the boundary layer equations become

$$\xi u\frac{\partial u}{\partial \xi}+\overline{V}\frac{\partial u}{\partial n}+\frac{\xi}{r^3}w\frac{\partial u}{\partial \overline{\phi}}-\frac{\xi r'}{r^3}w^2=\frac{-\xi}{\rho}\frac{\partial p_1}{\partial \xi}+\frac{u_\infty}{2}\frac{\partial^2 u}{\partial n^2}\quad\text{(x--Momentum)}$$

where

$$\overline{V}=\overline{v}+nu\left[\frac{\xi r'}{r^3}-\frac{1}{2}\right]-\frac{w\xi}{r^3}\frac{n}{2p_1}\frac{\partial p_1}{\partial \phi}-u\xi\frac{n}{2p_1}\frac{\partial p_1}{\partial \xi}$$

$$\xi\, u \frac{\partial w}{\partial \xi} + \bar{V} \frac{\partial w}{\partial n} + \frac{\xi}{r^3} w \frac{\partial w}{\partial \phi} + \frac{\xi r'}{r^3} uw = \frac{u_\infty}{2} \frac{\partial^2 w}{\partial n^2} - \frac{\xi}{r^3} \frac{1}{\rho} \frac{\partial p_1}{\partial \phi} \qquad (\phi\text{--Momentum})$$

$$\xi\, u \frac{\partial T}{\partial x} + \bar{V} \frac{\partial T}{\partial n} + \frac{\xi}{r^3} w \frac{\partial T}{\partial \phi} = \frac{1}{\rho c_p} \left[\xi\, u \frac{\partial p_1}{\partial \xi} + \frac{\xi}{r^3} w \frac{\partial p_1}{\partial \phi} \right]$$

$$+ \frac{u_\infty}{2c_p} \left[\left(\frac{\partial u}{\partial n}\right)^2 + \left(\frac{\partial w}{\partial n}\right)^2 \right] + \frac{u_\infty}{2Pr} \frac{\partial^2 T}{\partial n^2} \qquad (\text{Energy})$$

$$\xi \frac{\partial u}{\partial \xi} + \frac{\partial u}{\partial n} \left[- \frac{\xi n}{p_1} \frac{\partial p_1}{\partial \xi} - \frac{n}{2} + \frac{n \xi r'}{r^3} \right] + \frac{\partial \bar{V}}{\partial n} + \frac{\xi}{r^3} \frac{\partial w}{\partial \phi}$$

$$- \frac{n}{2p_1} \frac{\xi}{r^3} \frac{\partial w}{\partial n} \frac{\partial p_1}{\partial \phi} = - \frac{\xi r'}{r^3} u \,. \qquad (\text{Continuity})$$

The primary difference between these equations and those for a right circular cone are the additional terms involving axial pressure gradients, $\partial p_1/\partial \xi$. The axial pressure gradients are calculated from the inviscid flow equations, which have been solved by the "shock capturing" finite difference scheme. A problem which appeared was that the inviscid limit of the boundary layer equations was differenced in a slightly different method than the inviscid flow calculation. When this occurs the boundary layer equations can exhibit overshoots and undershoots as the inviscid flow is approached from the wall. However, if the inviscid pressure gradients are calculated with a procedure consistent with the boundary layer finite difference scheme, no difficulties arise.

III. TURBULENT FLOW AND TRANSPORT PROPERTIES

In order to form the boundary layer equations for turbulent flow, the dependent variables are broken up into fluctuating and mean components, reference 3. The result of this procedure yields the following equations in the physical plane.

$$\frac{\partial}{\partial x} (\bar{\rho}\, \bar{u}\, r) + \frac{\partial}{\partial y} (\bar{\rho}\, \tilde{V}\, r) + \frac{\partial}{\partial \phi} (\bar{\rho}\, \bar{w}) = 0$$

where

$$\tilde{V} = \bar{v} + \frac{\overline{\rho' v'}}{\bar{\rho}}$$

and the quantities with a bar on them imply an average.

3. *J. C. Adams, "Finite-Difference Analysis of the Three-Dimensional Turbulent Boundary Layer on a Sharp Cone at Angle of Attack in a Supersonic Flow," AIAA Paper No. 72-186, 10th Aerospace Sciences Meeting, San Diego, California, January 1972.*

$$\bar{\rho}\,\bar{u}\frac{\partial\bar{u}}{\partial x} + \bar{\rho}\,\tilde{V}\frac{\partial\bar{u}}{\partial y} + \frac{\bar{\rho}\,\bar{w}}{r}\frac{\partial\bar{u}}{\partial\phi} - \frac{\bar{\rho}\,\bar{w}^2}{r}\,r' = -\frac{\partial\bar{p}_1}{\partial x} + \frac{\partial}{\partial y}\left[\mu\frac{\partial\bar{u}}{\partial y} - \bar{\rho}\,\overline{u'v'}\right]$$

(x--Momentum)

$$\bar{\rho}\,\bar{u}\frac{\partial\bar{w}}{\partial x} + \bar{\rho}\,\tilde{V}\frac{\partial\bar{w}}{\partial y} + \frac{\bar{\rho}\,\bar{w}}{r}\frac{\partial\bar{w}}{\partial\phi} + \frac{\bar{\rho}\,\bar{u}\,\bar{w}}{r}\,r' = -\frac{1}{r}\frac{\partial\bar{p}_1}{\partial\phi} + \frac{\partial}{\partial y}\left[\mu\frac{\partial\bar{w}}{\partial y} - \bar{\rho}\,\overline{v'w'}\right]$$

(ϕ--Momentum)

$$\bar{\rho}\,\bar{u}\frac{\partial\bar{h}}{\partial x} + \bar{\rho}\,\tilde{V}\frac{\partial\bar{h}}{\partial y} + \frac{\bar{\rho}\,\bar{w}}{r}\frac{\partial\bar{h}}{\partial\phi} = \bar{u}\frac{\partial\bar{p}_1}{\partial x} + \frac{\bar{w}}{r}\frac{\partial\bar{p}_1}{\partial\phi} + \mu\left[\left(\frac{\partial\bar{u}}{\partial y}\right)^2 + \left(\frac{\partial\bar{w}}{\partial y}\right)^2\right]$$

$$-\bar{\rho}\,\overline{u'v'}\frac{\partial\bar{u}}{\partial y} - \bar{\rho}\,\overline{v'w'}\frac{\partial\bar{w}}{\partial y} + \frac{\partial}{\partial y}\left[\frac{\mu}{Pr}\frac{\partial\bar{h}}{\partial y} - \bar{\rho}\,\overline{v'h'}\right] \quad \text{(Energy)}$$

In order to reduce the number of unknowns in the equations, an eddy viscosity and a turbulent Prandtl number are introduced. These quantities are defined by the following relationships

$$\bar{\rho}\,\overline{u'v'} = \bar{\rho}\,\overline{v'w'} = \bar{\rho}\,\ell^2\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] = \bar{\rho}\,\varepsilon\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]^{1/2}$$

$$\bar{\rho}\,\overline{v'h'} = \frac{\varepsilon}{Pr_T}\,.$$

If the coordinate transformation plus these definitions are introduced into the boundary layer equations, they become

$$\xi\bar{u}\frac{\partial\bar{u}}{\partial\xi} + V\frac{\partial\bar{u}}{\partial n} + \frac{\xi}{r^3}\,\bar{w}\frac{\partial\bar{u}}{\partial\phi} - \xi\frac{r'}{r^3}\,\bar{w}^2 = \frac{u_\infty}{2}\frac{\partial}{\partial n}\left[\mu_T\frac{\partial\bar{u}}{\partial n}\right] - \frac{\xi}{\bar{\rho}}\frac{\partial\bar{p}_1}{\partial\xi}$$

(x--Momentum)

where

$$\mu_T = \frac{T_\infty}{T}\,\frac{(\mu+\varepsilon)}{\mu_\infty}$$

and

$$V = \tilde{V} + n\bar{u}\left[\frac{\xi r'}{r^3} - \frac{1}{2}\right] - \frac{\bar{w}\xi}{r^3}\frac{n}{2\bar{p}_1}\frac{\partial\bar{p}_1}{\partial\bar{\phi}} - \frac{\bar{u}\xi n}{2\bar{p}_1}\frac{\partial\bar{p}_1}{\partial\xi}$$

$$\xi\bar{u}\frac{\partial\bar{w}}{\partial\xi} + V\frac{\partial\bar{w}}{\partial n} + \frac{\xi\bar{w}}{r^3}\frac{\partial\bar{w}}{\partial\phi} + \frac{\xi r'\bar{u}\,\bar{w}}{r^3} = -\frac{\xi}{r^3}\frac{1}{\bar{\rho}}\frac{\partial\bar{p}_1}{\partial\bar{\phi}} + \frac{u_\infty}{2}\frac{\partial}{\partial n}\left[\mu_T\frac{\partial\bar{w}}{\partial n}\right]$$

(φ--Momentum)

$$\xi\bar{u}\frac{\partial\bar{T}}{\partial\xi} + V\frac{\partial\bar{T}}{\partial n} + \frac{\xi}{r^3}\bar{w}\frac{\partial\bar{T}}{\partial\bar{\phi}} = \frac{1}{\bar{\rho}c_p}\left[\xi\bar{u}\frac{\partial\bar{p}_1}{\partial\xi} + \frac{\xi\bar{w}}{r^3}\frac{\partial p_1}{\partial\bar{\phi}}\right]$$

$$+ \frac{u_\infty}{2}\frac{\mu_T}{c_p}\left[\left(\frac{\partial\bar{u}}{\partial n}\right)^2 + \left(\frac{\partial\bar{w}}{\partial n}\right)^2\right] + \frac{u_\infty}{2}\frac{\partial}{\partial n}\left[\frac{T_\infty}{\bar{T}}\,\frac{\mu/Pr + \varepsilon/Pr_T}{\mu_\infty}\,\frac{\partial\bar{T}}{\partial n}\right]$$

(Energy)

$$\xi\frac{\partial\bar{u}}{\partial\xi} + \frac{\partial\bar{u}}{\partial n}\left[-\frac{\xi n}{2p_1}\frac{\partial p_1}{\partial\xi} - \frac{n}{2} + \frac{n\xi r'}{r^3}\right] + \frac{\partial\tilde{V}}{\partial n} + \frac{\xi}{r^3}\frac{\partial\bar{w}}{\partial\bar{\phi}} - \frac{n}{2p_1}\frac{\xi}{r^3}\frac{\partial\bar{w}}{\partial n}\frac{\partial p_1}{\partial\bar{\phi}} = \frac{\xi r'\bar{u}}{r^3}.$$

(Continuity)

The one further step to be taken is to introduce a semi-empirical formula for the mixing length, ℓ, appearing in the expression for the eddy viscosity. The expression used was taken from reference 3 and is

$$\ell = \lambda\,\delta\,\tanh\left(\frac{.4}{\lambda}\frac{y}{\delta}\right)$$

where $\lambda = .09$ and δ is the boundary layer thickness defined by

$$y = \delta \quad \text{at } \frac{u}{u_1} = .995 .$$

In order to efficiently calculate a turbulent boundary layer by a numerical method, one additional coordinate transformation must be applied. This transformation allows for close grid spacing near the wall and wider spacing in the outer regions of the boundary layer. The transformation is described in reference 4 and is

$$n_j = n_\delta(K^{N_j/\Delta N_o} - 1) \quad (K^{1/\Delta N_o} - 1) \qquad j = 1, 2, 3, \ldots, J$$

where n_δ is the value of n at the boundary layer edge, j is the n finite difference index, $N_j = (j - 1)\Delta N$ and $N_J = 1$. The values of K and ΔN_o

4. *F. G. Blottner, "Variable Grid Scheme Applied to Turbulent Boundary Layers," Journal of Computer Methods in Applied Mechanics and Engineering, 1975.*

are two parameters which are chosen to give the desired grid spacing. Acceptable values for the turbulent boundary layers occurring over artillery projectiles are in the range $K \cong 1.5$ and $\Delta N_o = .05$. The changes required in the boundary layer equations are that all derivatives with respect to n become

$$\frac{\partial}{\partial n} = \frac{\partial}{\partial N} \frac{\partial N}{\partial n}$$

where $\partial N/\partial n$ is evaluated using finite differences from the defining relationship between n and N given above.

The finite difference methods used to integrate the turbulent boundary layer equations was the same as reference 2. A modification developed not previously discussed was the use of a damping function to make the transition from the fully turbulent flow to the laminar sublayer near the wall. The damping will be denoted by D and is given by

$$D = 1 - \exp(-y^+/A^+)$$

where the notation is described in reference 2. To apply this correction, the mixing length ℓ is multiplied by D. One further semi-empirical modification for turbulence was also introduced and consisted of a function to make the transition from laminar to turbulent flow. The one chosen was simply a linear increase of ε from 0 to its full value over four grid points. This modification worked very satisfactorily in the absence of appropriate data on transition.

IV. CALCULATION OF THE THREE-DIMENSIONAL DISPLACEMENT THICKNESS

In order to calculate the three-dimensional displacement thickness Δ for flow over an artillery projectile, a new equation had to be derived. This new equation reflects the differences between right circular cones and artillery projectiles, and is different than the one used by reference 5. The new equation is

$$\rho_1 u_1 r \frac{\partial \Delta}{\partial x} + \rho_1 w_1 \frac{\partial \Delta}{\partial \phi} = -\Delta \left[\frac{\partial}{\partial x} [\rho_1 u_1 r] + \frac{\partial}{\partial \phi} [\rho_1 w_1] \right] + \frac{\partial}{\partial x} (\rho_1 u_1 r \delta_x) + \frac{\partial}{\partial \phi} (\rho_1 w_1 \delta_\phi) \ .$$

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5. *B. R. Sanders, "Three-Dimensional, Steady, Inviscid Flow Field Calculations with Application to the Magnus Problem," Ph.D. Thesis, Department of Mechanical Engineering, University of California-Davis, California, 1974.*

where δ_x and δ_ϕ are the primary flow and cross flow displacement thicknesses, respectively.

It was also found that the method of solution used in reference 5 had an error in it, and a new method had to be developed. The new method uses backward differences for $\partial\Delta/\partial x$ and $\partial\Delta/\partial\phi$ and obeys the single mathematical characteristic. This characteristic is defined by the equation

$$\frac{dx}{d\phi} = \frac{ru_1}{w_1} .$$

The new method has been found satisfactory for the small angle of attack cases considered up until the present time.

V. BOUNDARY REGION EQUATIONS

At large angle of attack or whenever the cross flow velocity "separates," the boundary layer equations as described above are not an adequate description of the viscous flow near the wall. In order to describe this flow, the boundary region equations have been proposed, reference 6. The boundary region equations for an artillery projectile are listed below. These equations contain additional terms from the complete Navier-Stokes equations, which represent viscous effects in the cross flow direction. The equations are as follows

$$\frac{\partial}{\partial x}(\rho r u) + \frac{\partial}{\partial y}(\rho r v) + \frac{\partial}{\partial \phi}(\rho w) = 0 \qquad \text{(continuity)}$$

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{w}{r}\frac{\partial u}{\partial \phi} - \frac{w^2 r'}{r}\right] = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left[\mu\frac{\partial u}{\partial y}\right] + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left[\mu\frac{\partial u}{\partial \phi}\right]$$

$$+ \frac{\mu}{r}\frac{\partial u}{\partial y}\frac{\partial r}{\partial y} \qquad \text{(x--Momentum)}$$

$$\rho\left[u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + \frac{w}{r}\frac{\partial w}{\partial \phi} + \frac{uwr'}{r}\right] = -\frac{1}{r}\frac{\partial p}{\partial \phi} + \frac{\partial}{\partial y}\left[\mu\frac{\partial w}{\partial y}\right] + \frac{4x}{3r^2}\frac{\partial}{\partial \phi}\left[\mu\frac{\partial w}{\partial \phi}\right]$$

$$+ \frac{\mu}{r}\frac{\partial w}{\partial y}\cos\theta \qquad (\phi\text{--Momentum})$$

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6. *T. C. Lin and S. G. Rubin, "A Two-Layer Model for Coupled Three-Dimensional Viscous and Inviscid Flow Calculations," AIAA Paper No. 75-853, 8th Fluid and Plasma Dynamics Conference, Hartford, Connecticut, June 1975.*

$$\rho c_p \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \frac{w}{r} \frac{\partial T}{\partial \phi} \right] = u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + \frac{\partial}{\partial y} \left[k \frac{\partial T}{\partial y} \right] + \mu \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] + \frac{k}{r} \frac{\partial T}{\partial y} \cos\theta + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) + \frac{4}{3} \frac{\mu}{r^2} \left(\frac{\partial w}{\partial \phi} \right)^2 + \frac{\mu}{r^2} \left(\frac{\partial u}{\partial \phi} \right)^2 + \mu \frac{w^2}{r^2} \cos^2 \phi \; . \qquad \text{(Energy)}$$

The solution of these equations is beyond the scope of the present task, but their solution will be necessary to fully understand the Magnus problem on artillery projectiles.

VI. RESULTS AND SUMMARY

It can be said that this research effort has been very successful. The major modifications of the cone results such as axial pressure gradient, turbulent flow and three-dimensional displacement thickness have been completed. Dr. Walter Sturek and Dr. Robert Reklis of BRL are using computer programs written by the present author and these computer programs are calculating the Magnus force over artillery-type projectiles for realistic conditions. Dr. Sturek is comparing these results with preliminary experimental data, and the agreement looks very good.

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1. H. A. Dwyer, "Three Dimensional Flow Studies Over a Spinning Cone at Angle of Attack," BRL Contract Report No. 137, U.S. Army Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland, February 1974. AD 774795.

2. H. A. Dwyer and B. R. Sanders, "Magnus Forces on Spinning Supersonic Cones, Part I: The Boundary Layer," *AIAA Journal*, Vol. 14, No. 4, April 1976, pp. 498-504.

3. J. C. Adams, "Finite-Difference Analysis of the Three-Dimensional Turbulent Boundary Layer on a Sharp Cone at Angle of Attack in a Supersonic Flow," AIAA Paper No. 72-186, 10th Aerospace Sciences Meeting, San Diego, California, January 1972.

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LIST OF SYMBOLS

c_p	specific heat at constant pressure
h	static enthalpy
k	molecular conductivity
ℓ	mixing length
n	transformed y coordinate
p	pressure
P_r	molecular Prandtl number, $c_p \mu / k = .71$
r	local radius of model
u	velocity component in x direction
v	velocity component in y direction
w	velocity component in ϕ direction
x	surface coordinate in longitudinal direction
y	coordinate perpendicular to local surface
ε	turbulent eddy viscosity
δ	boundary layer thickness
δ^*	boundary layer displacement thickness
Δ	three dimensional boundary layer displacement thickness
μ	molecular viscosity
ξ	transformed x coordinate
ρ	density
ϕ	coordinate in circumferential (azimuthal) direction

LIST OF SYMBOLS (Continued)

Subscripts

1 edge of boundary layer

∞ free stream reference condition

Superscripts

' fluctuating quantity

- time averaged quantity

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